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AFGL-TR-88-0141



# Lg-Wave Propagation in Heterogeneous Media

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6 April 1988

Scientific Report No. 1

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AIR FORCE GEOPHYSICS LABORATORY
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HANSCOM AIR FORCE BASE, MASSACHUSETTS 01731-5000

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

# REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3. DISTRIBUTION / AVAILABILITY OF REPORT: Approved for Public Release Distribution Unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

5. MONITORING ORGANIZATION REPORT NUMBER(S): AFGL-TR-88-0141

6a. NAME OF PERFORMING ORGANIZATION: Research School of Earth Sciences

6b. OFFICE SYMBOL (If applicable)

7a. NAME OF MONITORING ORGANIZATION: Air Force Geophysics Laboratory

6c. ADDRESS (City, State, and ZIP Code): Australian National University, GPO Box 4, CANBERRA ACT 2601, Australia

7b. ADDRESS (City, State, and ZIP Code): Hanscom AFB, Massachusetts 01731-5000

8a. NAME OF FUNDING / SPONSORING ORGANIZATION: Air Force Geophysics Laboratory

8b. OFFICE SYMBOL (If applicable)

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: Grant AFOSR-87-0187

8c. ADDRESS (City, State, and ZIP Code): Hanscom AFB, Massachusetts 01731-5000

10. SOURCE OF FUNDING NUMBERS

| PROGRAM ELEMENT NO. | PROJECT NO. | TASK NO | WORK UNIT ACCESSION NO. |
|---|---|---|---|
| 61101E | 7A10 | DA | CB |

11. TITLE (Include Security Classification): Lg-wave Propagation in Heterogeneous Media (Unclassified)

12. PERSONAL AUTHOR(S): KENNETT, Brian L.N.

13a. TYPE OF REPORT: Scientific #1

13b. TIME COVERED: FROM 87/2/15 TO 88/2/14

14. DATE OF REPORT (Year, Month, Day): 1988 April 6

15. PAGE COUNT: 44

16. SUPPLEMENTARY NOTATION: This research was supported by the Defense Advanced Research Projects Agency

17. COSATI CODES

| FIELD | GROUP | SUB-GROUP |
|---|---|---|
| | | |
| | | |

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number): Wave Propagation; Lg waves; Heterogeneity; Coupled Modes

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The Lg wave phase which is of considerable interest for nuclear discrimination problems is normally observed after propagation through considerable distances. This phase is dominantly guided in the crustal waveguide, which is known to be a region with very considerable horizontal variability in properties.

The effect of heterogeneous crustal structures on Lg waves has been determined by using a "coupled-mode" technique in which the local seismic wavefield in the real medium is expressed as a combination of the modal eigenfunctions of a stratified reference structure. Departures of the seismic properties in the medium from those of the reference medium lead to coupling between the amplitude coefficients in the modal expansion. The evolution of these modal weighting factors with horizontal position are described by a coupled set of ordinary differential equations.

(Continued on reverse)

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT: ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION: Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL: James F. Lewkowicz

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22c. OFFICE SYMBOL: LWH

DD Form 1473, JUN 86 Previous editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

19. ABSTRACT (continued)

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A sequence of models with varying levels of heterogeneity have been considered in order to determine the merits and limitations of the computation scheme. The coupled mode technique works well with heterogeneous models in which the local seismic velocities differ from the stratified reference model by up to 2 per cent and there are no significant distortions of the main discontinuities (e.g. the crust-mantle boundary). The approach can be used for higher levels of heterogeneity and with distorted interfaces but a large number of modes needs to be considered with consequent high computation costs. If the level of heterogeneity is not too large then the interaction between modes can be restricted, rather than extending over the whole mode set, with consequent reduction in computation cost.

One of the major effects of crustal heterogeneity is to introduce the possibility of smearing out the main amplitude peak in the Lg wave train over a band of group velocities. As a result, an effective measure of the energy content of the Lg waves will be to consider the integrated amplitude along the traces beween group velocities of 3.6 and 3.3 km/s. The effects of heterogeneity vary between different parts of the Lg wave train and the representation of the wavefield in terms of modal contributions allows a detailed analyis in terms of the group velocity components.

The computational scheme is currently being extended to try to allow for mode-coupling in three-dimensions for both Love and Rayleigh components of Lg waves. When heterogeneity transverse to the propagation path is relatively weak a good approximation to the propagation characteristics can be produced.

## SUMMARY


The Lg wave phase which is of considerable interest for nuclear discrimination problems is normally observed after propagation through considerable distances. This phase is dominantly guided in the crustal waveguide, which is known to be a region with very considerable horizontal variability in properties.

The effect of heterogeneous crustal structures on Lg waves has been determined by using a "coupled-mode" technique in which the local seismic wavefield in the real medium is expressed as a combination of the modal eigenfunctions of a stratified reference structure. Departures of the seismic properties in the medium from those of the reference medium lead to coupling between the amplitude coefficients in the modal expansion. The evolution of these modal weighting factors with horizontal position are described by a coupled set of ordinary differential equations.

This approach provides a calculation scheme for studying guided wave propagation over extended distances, at frequencies of 1 Hz and above. The heterogeneity models which have been used are two-dimensional and calculations are carried out for one frequency at a time.

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The computational scheme is currently being extended to try to allow for mode-coupling in three-dimensions for both Love and Rayleigh components of Lg waves. When heterogeneity transverse to the propagation path is relatively weak a good approximation to the propagation characteristics can be produced.

CONTENTS

## RESEARCH OBJECTIVES

The aim of this work is to develop techniques for handling guided wave propagation in three-dimensionally heterogeneous media, especially for Lg waves, in order to improve the understanding of the nature of regional seismic phases and the way in which the characteristics of the seismic source can be modified by propagation to the receiver. Such information will be valuable in assessing the effect of geological structure on the behaviour of potential discriminants between earthquakes and underground nuclear explosions.

Most of the regional phases of interest for nuclear discrimination problems are observed after propagation through considerable distances. These phases travel through the crust and the uppermost mantle, which from a variety of studies are known to be regions with very considerable horizontal variability in properties.

What is therefore needed is a computational procedure which will allow the tracking of guided seismic wave propagation through a horizontally heterogeneous crust for distances up to 1000 km or more. Such a method should be able to be used with three-dimensional variations in seismic properties superimposed on the normal increase in seismic wave velocities with depth. A suitable candidate is the coupled mode scheme described by Kennett (1984) which has already been used with some success in understanding the characteristics of Lg wave behaviour (Kennett & Mykkeltveit 1984).

## RESEARCH STATUS AND ACHIEVEMENTS

During the past year, the work on Lg phases has mostly been carried out using the coupled modes method introduced by Kennett (1984). Firstly, the limitations of the calculation scheme have been explored for two-dimensional heterogeneity models and a number of approximations have been explored with the aim of improving computational performance. Secondly, an attempt has been made to extend the methods to more general three-dimensional models of heterogeneity.

For regional S wave trains, a representation in terms of a limited number of discrete modes gives a economical computational description for horizontally stratified media. At each frequency only a limited number of such modes need be considered. In a laterally heterogeneous medium, at a single frequency, it is possible to represent the seismic displacement and traction fields within the varying medium as a sum of contributions from the modal eigenfunctions of a reference structure, with coefficients which vary with position. The evolution of these modal expansion coefficient terms with horizontal position can be described by a set of coupled partial differential equations in the horizontal coordinates (an outline of the theoretical development is given for completeness in the Appendix). The cross-coupling terms between different modes depend on the departures of the heterogeneous structure from the stratified reference model. These differences are not required to be very small but must not be such as to completely change the nature of the crustal wave guide. Thus, it is possible to accommodate substantial localised change in seismic velocities and density, but more difficult to allow for shifts of more than 2-3 km in the position of major interfaces such as the crust-mantle boundary. Maupin & Kennett (1987) give an extended discussion of the circumstances in which the coupled mode approach can be applied to two-dimensional heterogeneous models.

### *Studies of Two-dimensional Heterogeneity in the Crust and Mantle*

When the heterogeneity within the medium is two-dimensional, the calculations can be recast as the solution of non-linear differential equations of Ricatti equation for the reflection and transmission matrices connecting the modal expansion coefficients at different positions. The advantage of this rearrangement is that, for each frequency, the boundary conditions on the differential equations are simplified for a generally heterogeneous medium.

If we adopt a reference medium which does not vary with horizontal position, the individual mode contributions propagate independently in that structure. However, once the properties of the true medium differ from the reference, the independence of modal propagation is lost. The effect of the heterogeneity enters into the differential equations for the modal expansion coefficients via a coupling matrix whose dimensions are dictated by the number of modes included in the calculation.

The choice of the number of modes is of considerable importance. All the significant wavenumber components for the seismic phase of interest should be included, as well as an allowance for steeper angles of propagation than would be present in the reference medium, in order to allow for scattering effects. The demands of computation are that the number of modes should be kept as small as possible, since the computation time depends on the square of the number of modes. On the other hand if the truncation is too tight, a poor representation of the displacement field can ensue and the results are of limited use.

In figure 1 we illustrate the reference model (ARANDA) used for most of the Lg mode calculations, and the modal eigenfunctions for the 22 Love modes included in calculations at 1.5 Hz. The cut-off criterion was based on the effective depth of penetration of the modes. The fundamental mode and first 15 higher modes are confined to the crust, and are the main contributors to Lg. Mode 16 begins to have significant displacement in the mantle, and the remaining modes carry their main energy in the mantle and represent the Sn phase. The truncation is made at a phase velocity of 4.54 km/s, corresponding in this model to a penetration depth of about 100 km. Features in the wave propagation process corresponding to higher phase velocities than 4.54 km/s cannot be represented by these coupled mode calculations.

In order to give a good account of the guided wave propagation, we must make sure that all the significant wavenumber components for the seismic phase of interest are included, as well as making an additional allowance for steeper angles of propagation in order to allow for scattering effects. This means that we cannot just use those modes which have the bulk of their energy within the crust but must also include waves with higher phase velocity whose energy in the stratified reference model lies dominantly in the upper mantle. At 1.5 Hz as shown in figure 1, the fundamental mode and first 15 higher modes are confined to the crust, and are the main contributors to Lg. However, we need to carry at least the 22 modes illustrated if we are to allow for wave interaction processes generating waves travelling at angles up to 40° from the horizontal in the midcrust and even then we will not give a full account of waves travelling at steeper angles to the horizontal.

An indication of the way in which the coupled mode scheme is likely to behave when confronted with heterogeneity is provided by looking at the expansion of the modal eigenfunctions of a particular stratified structure in terms of the eigenfunctions of a different reference structure. The situation is illustrated in figure 2 for two cases: firstly, where the crust-mantle interface is maintained fixed but the crustal velocity is varied, and secondly when the crust-mantle interface is moved by 1 km. The pattern of modal behaviour is rather different in the two situations. For a change in crustal velocity (fig 2a), the modal eiigenfunction shapes remain similar in the varied model and so the expansion coefficents are largest for the original mode and its immediate neighbours and decay quite rapidly with increasing mode separation. A 1 km shift in the crust-mantle bounary has a more pronounced effect (fig 2b), now the modal expansion coefficients alternate in sign and decay less rapidly with increasing mode separation. This arises because the structure of the dispersion equations and therefore, the nature of the modal eigenfunctions, is much more sensitive to the position of a major discontinuities than to the details of the velocity distribution between them. A consequence is that for a perturbed interface many modes are needed in a modal expansion to give an accurate representation of the displacement and traction fields.

Figure 1: ARANDA reference model and Love mode eigenfunctions with phase velocity less than 4.54 km/s at frequency 1 Hz.

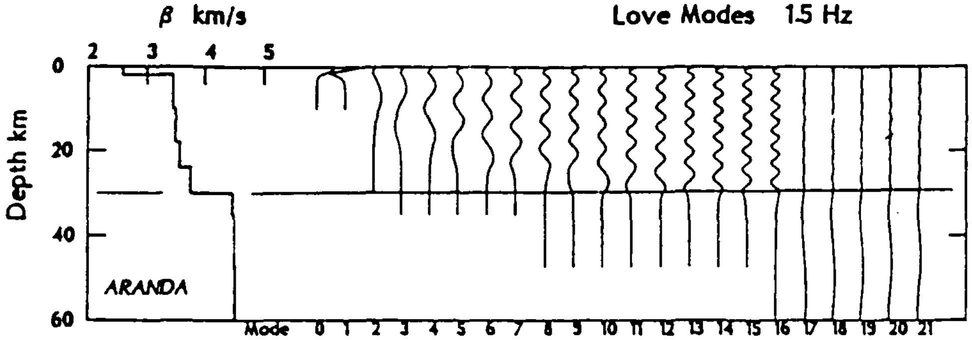
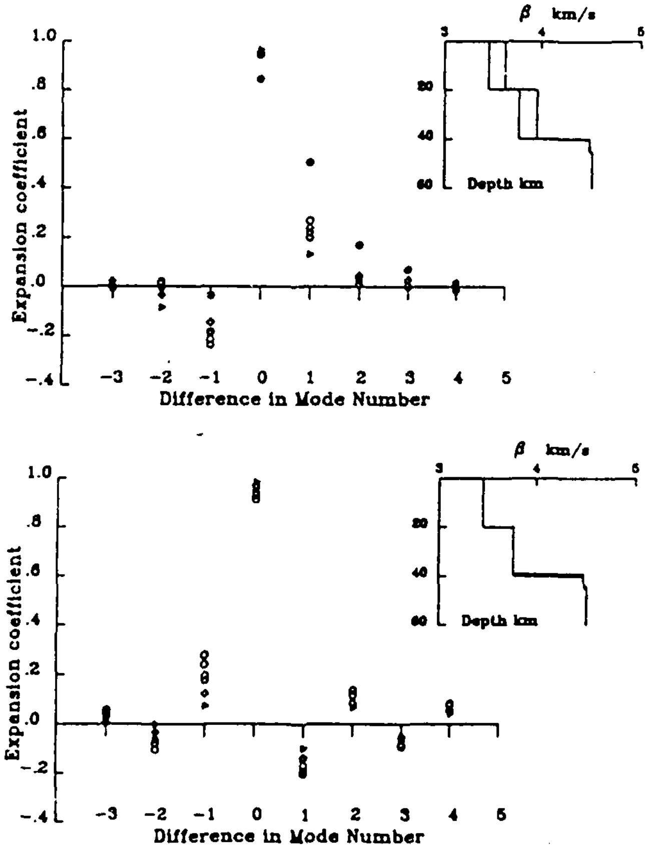



Figure 2: Illustration of the expansion of the eigenfunctions of a varied model in terms of those for a reference structure. The expansion coeffiecients are shown for a span of 8 modes. The reference model and the varied model, indicated by the chain dotted line, are shown in the insert.
a) change in velocity, b) change in depth to crust-mantle boundary



Significant perturbations of the major interfaces in the model thus lead to extensive coupling between modes and can induce relatively steep angles of propagation. If such features are required in the models to be considered, then a broad sweep of modes must be taken for the reference model (for the Lg case at 1.5 Hz, at least 30). With the benefit of hindsight, it would appear that the calculations of Kennett & Mykkeltveit (1984) on the influence of graben structures lie at the limit of acceptable model variations for the modal set adopted.

The coupled mode representation is well suited to heterogeneous models where the behaviour consists of relatively random variations about the properties of the reference model (see e.g. fig 3). In this case, a single incident mode will interact with a limited number of neighbouring modes so that the coupling matrix is diagonal dominated with a limited effective bandwidth.

With the aid of the coupled mode technique, a study has been made of the way in which the modal field for Lg waves is affected by various levels of distributed heterogeneity for frequencies up to 2Hz over paths of 1000 km long. A sequence of different heterogeneity models were constructed by specifying the seismic velocities in a vertical section at 40 km intervals and then using bicubic spline interpolation within each layer. For the distributed heterogeneity models the velocity values were generated with a random perturbation to the reference value within a prescribed range of variation. The velocity values were then smoothed horizontally by applying a moving average over 3 points. This imposes a typical horizontal scale length of around 100 km, and the vertical scale varies within the model becoming larger in the upper mantle. The velocity distribution is arranged to return to the reference structure at the ends of the model (0 and 1000 km) so that a direct physical interpretation of the node coupling results in terms of reflection and transmission can be made. In figure 3 we illustrate a set of models (D, E, F) with increasing crustal heterogeneity. Model D is constructed to have about $\pm 1$ per cent heterogeneity in the crust with a horizontal scale length of about 100 km, and $\pm 1$ per cent heterogeneity has been introduced in the mantle as well. Models E and F retain the same mantle heterogeneity structure as model D but have higher crustal heterogeneity amplitudes, $\pm 2$ per cent for model E and $\pm 5$ per cent for model F. We shall also consider model C wi the same crustal structure as D, but no heterogeneity below the crust-mantle boundary.

After propagating a large horizontal distance in a heterogeneous model the energy originally in a single incident mode will no longer be confined to that mode, and indeed some energy may be reflected back by the heterogeneity, with again the possibility of conversion between modes. When the level of heterogeneity is relatively low (of the order of 1 per cent deviations from the stratified reference model - as in model D) transmission effects dominate and the level of reflected energy is negligible. Even with rather larger levels of heterogeneity the reflection from the whole model is small but, as pointed out by Maupin & Kennett (1987), it is necessary to retain apparently reflected waves in the course of the computation in order to get an accurate calculation of transmission effects. The Ricatti equations are much simpler when reflection can be neglected and so computation time can be reduced.

For any particular structure, it is therefore necessary to undertake a preliminary calculation including both reflection and transmission effects before it can be assumed that reflection can be ignored. The control of numerical accuracy in the course of the calculations is rather difficult, although for perfectly elastic models there is the possibility of monitoring the constancy of the total energy in reflection and transmission associated with an individual incident mode. To preserve full numerical precision quite small steps have to be taken in the horizontal direction: at 1.5 Hz the step should be no larger than 0.5 km if both reflected and transmitted waves are considered. For a number of models a somewhat larger step length gave reasonable results for the case of transmission alone.

Figure 3: Sequence of heterogeneous velocity models used in studies of Lg wave propagation. Heavier shading indicates negative perturbations away from the ARANDA reference model.

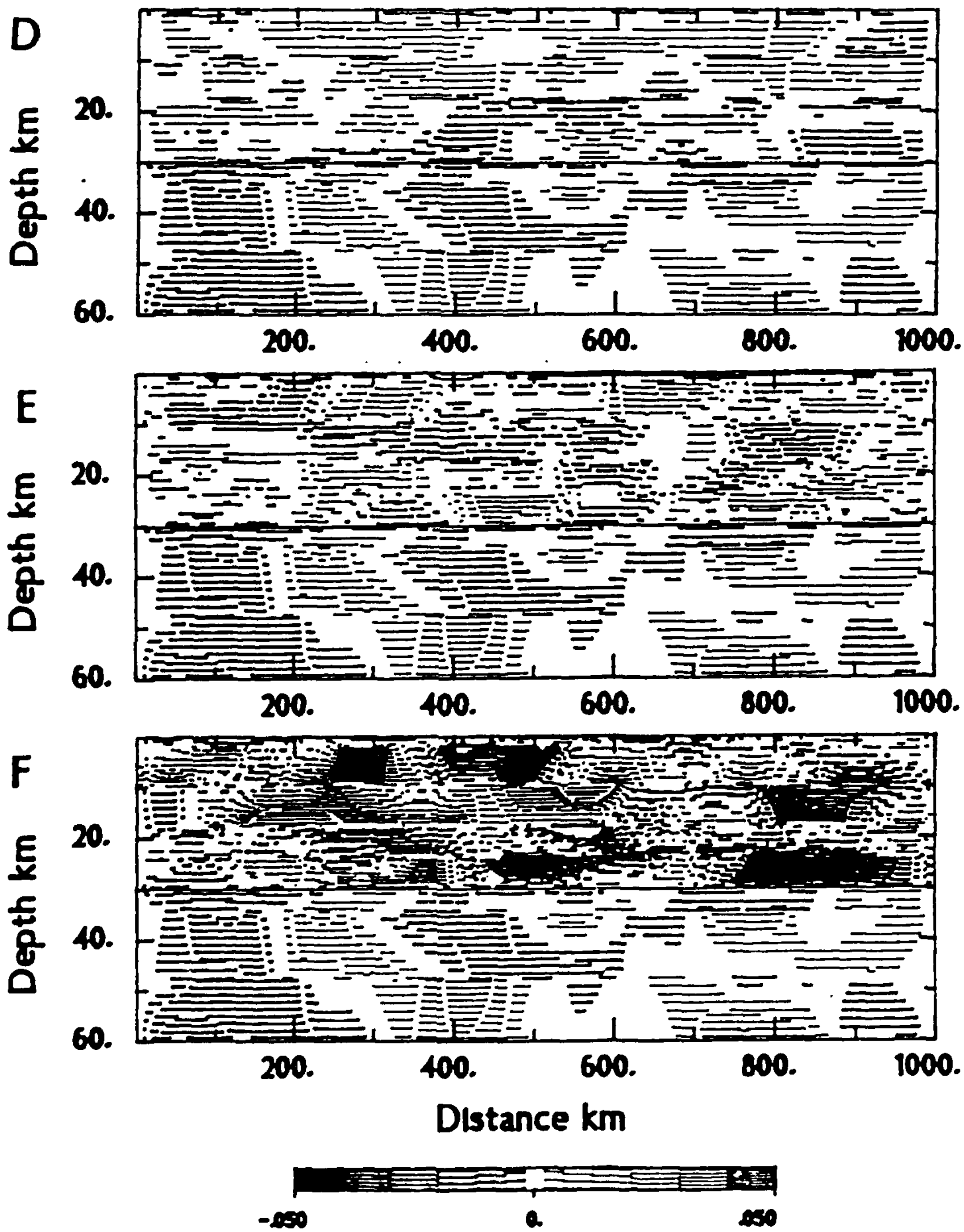
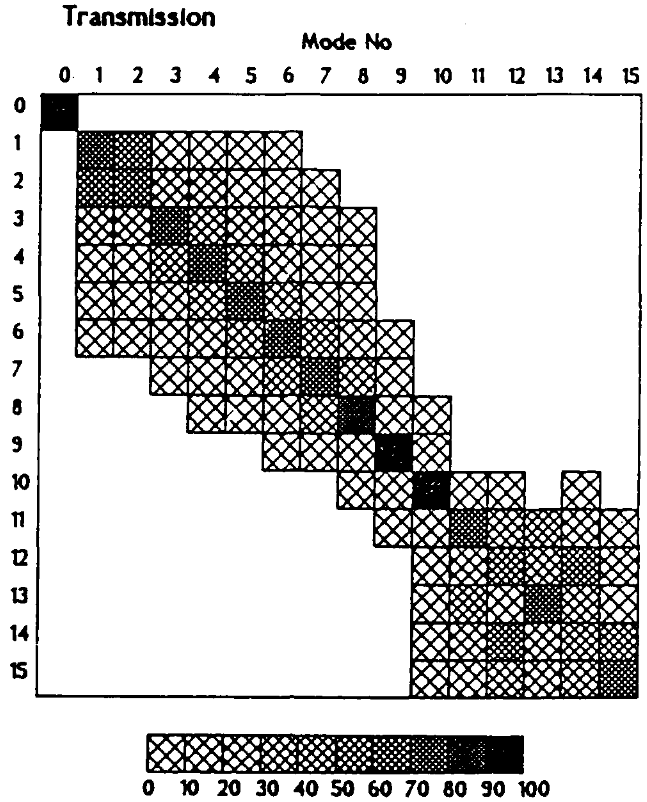


Figure 4: Representation of the transmission matrix at 1 Hz after passage through 1000 km of model D. Amplitude values less than 1 are left blank.




For an incident mode at 0 km the effect of 1000 km of heterogeneity is that the energy originally in the mode is no longer confined to that particular mode. In figure 4 we show the transmission matrix for the 15 mode set appropriate to the ARANDA reference model at 1 Hz, after passage through the heterogeneity model D. Each column of the matrix corresponds to the incidence of a single mode of amplitude 100 at 0 km. We see that the behaviour is diagonal dominated, but that significant mode conversions can occur. For the Lg type modes (0-9) the bandwidth for significant interaction is typically ± 2 modes. The strong interaction of modes 1 and 2 arises because of their similar shape in the top 10 km. For the Sn modes (11-15), the eigenfunctions are very similar and there is strong interaction due to mantle perturbation extending to 200 km (the lower portion is not shown in fig 3). However the separation from the Lg modes is striking; there is very little interaction except for mode 10 which shares some of the characteristics of each group.

As the frequency increases the number of modes need to cover the same phase velocity range increases, which causes some problems in combining the results from different frequencies. The increased size of the differential equation system also means an increase in computation time as the square of the number of modes if all mode interactions have to be considered. Fortunately, present tests indicate that for the same model the bandwidth of interaction increases only slowly with frequency. For the Lg waves propagating through models similar to D with about 1 per cent heterogeneity, computations can be restricted to a band of 5 modes either side of the target mode without appreciable error. At 1.5 Hz with 22 modes included, this bandwidth restriction has the effect of reducing computation time by nearly a factor of three.

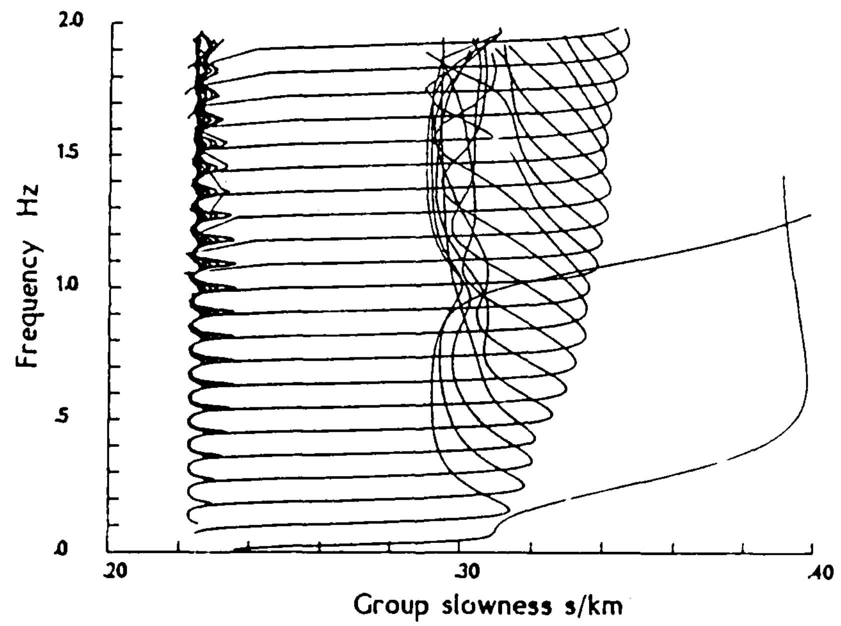
The required bandwidth increases with the level of heterogeneity in the model (see Table 1). For model D in figure 3, a bandwith of ±5 modes around the target mode is

sufficient for both Lg and Sn modes. However to account for the interaction between modes for model E it is desirable to have at least a bandwidth of ±7 modes included in the calculation. For the highest level of heterogeneity illustrated in fig 3 (model F) a bandwidth of ±9 modes is definitely insufficient for full accuracy. For very heterogeneous models there is also the possibility of high angle propagation effects and so the size of the mode set has to be increased. Thus although the coupled mode approach is not confined to low level heterogeneity, it is at is most effective when the structure does not deviate too far from the reference. A convenient working limit would seem to be about 2 per cent deviation as in model E.

In order to understand the effect of propagation through a heterogeneous model on the nature of regional phases we need to understand the characteristics of the surface wave modes in the reference model. In figure 5 we display the group slowness behaviour as a function of frequency up to 2 Hz for the first 40 Love modes on the ARANDA reference structure. The advantage of displaying the group slowness (the reciprocal of the group velocity) is that the various arrivals can be recognised in the time relationship that they would have on a seismic record.

The Sn phase can be recognised in figure 5 as the superposition of many slowness maxima and minima at a slowness around 0.22 s/km (4.5 km/s). All 40 modes contribute to this tightly defined band of slownesses which clearly separates from the other classes of arrival. The onset of the Lg phase at a slowness of 0.286 s/km (3.5 km/s) is associated with the superposition of a number of relatively broad slowness minima arising from relatively low order modes. Following this onset is a more tangled skein of maxima and minima with slownesses less than 0.304 s/km (i.e. group velocities greater than 3.3 km/s), associated with about four modes at each frequency, which will provide the main Lg arrival. Somewhat later, we have a sequence of sharply defined slowness maxima associated with each mode in turn as the frequency increases. These maxima represent the Airy phase for each mode and will be significant contributors to the Lg coda: they arise physically from multiple reflections at angles to the vertical just greater than the critical angle for the crust-mantle boundary.

Figure 5: Group slowness beviour as a function of frequency for the first 40 Love modes of the ARANDA reference model illustrated in figure 1.



From the modal eigenfunction patterns displayed in figure 1, we can recognise that those modes (3-6) which contribute most to the onset of Lg at 1.5 Hz have the bulk of their energy confined to the middle and upper crust. The main Lg arrivals come from modes 7-10 which sample the whole crust. The Airy phases for modes 14, 15 arise just before the transition to energy transport in the mantle.

The effect of varying the heterogeneity level in the structural models can be well illustrated by comparing the energy distribution across the modal sequence for a single incident mode. In Table 1 we show this energy distribution for a set of incident mode at 0 km, at a frequency of 1.5 Hz, chosen to represent different parts of the Lg wave phase. We consider horizontal transmission through 1000 km of structure for four different models C, D, E and F with increasing levels of heterogeneity. Model C has 1 per cent of hetrogeneity confined to the crust. Models D, E and F are illustrated in figure 3, and have the same heterogeneity model in the mantle with about 1 per cent variation from the reference model (ARANDA, fig 1), but increasing levels of crustal heterogeneity (±1 per cent for D, ±2 per cent for E, and ±5 per cent for F).

At 1.5 Hz, mode 4 is a principal contributor to the onset of the Lg wave train. From Table 1 we see that for the incidence of this single mode at 0 km, the majority of the energy is carried in the original mode over the full 1000 km propagation path for the structures with up to 2 per cent variation away from the reference model (C,D,E). However, the proportion of energy in the original mode decreases as the crustal heterogeneity increases. As expected the mantle heterogeneity has very little influence on this group of interacting modes which is principally confined to the crust. Once we reach the highest level of heterogeneity (model F) the pattern of the energy distribution is markedly changed, the energy maximum is shifted over two modes to mode 6 which has a slightly different group velocity and energy is spread widely across the whole suite of crustal modes. There is very little interaction with the fundamental or first higher modes because, as can be seen from figure 1, their energy is confined to the near surface and so is only sensitive to a small part of the total crustal heterogeneity.

For a mode in the main Lg wave arrivals (mode 9), we see that for the two models with relatively modest levels of heterogeneity (C,D) the energy is concentrated over three neighbouring modes centered on the incident mode. These three modes have group velocities varying by about 0.15 km/s and so the effect of the this level of heterogeneity will be to give a more diffuse maximum in the Lg wave amplitude. The presence of mantle heterogeneity now has a slight effect since it is possible to couple into modes with some energy penetration into the uppermost mantle. For the higher levels of crustal heterogeneity the incident energy gets spread out over a broader range of modes (6 are significant for model E and at least 10 for model F) and the effect on the Lg wavetrain will be more profound. For modest levels of heterogeneity (around 1 per cent) the best measure of Lg wave strength will be the integrated energy in a time window spanning group velocities of about 3.6-3.3 km/s for a typical continental situation.

Similar results arise for the Lg wave coda (incident mode 14) though this case, where the incident mode is most sensitive to lower crustal structure, is not as strongly affected by the heterogeneity as for mode 9. Even so, with increasing heterogeneity levels the representation of the propagation via the modes of the reference structure requires extensive mode coupling. For the higher heterogeneity levels there is significant energy shift into Sn type modes, which we can explain physically as occurring from local changes in the critical angle at the crust-mantle boundary. Modes like 14 travel close to the critical angle in the reference model and so a slight change of conditions can lead easily to energy transmission into the mantle.

Table 1: Energy distribution across the modes of the ARANDA model at 1.5 Hz, after transmission through 1000 km of heterogeneous structure

a) Incident Mode 4 (onset of Lg)

| Relative Mode Number: | | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| Model: | C | .000 | .032 | .048 | .689 | .109 | .078 | .003 | .023 |
| | D | .000 | .032 | .044 | .689 | .109 | .078 | .006 | .023 |
| | E | .000 | .040 | .212 | .563 | .032 | .036 | .084 | .002 |
| | F | .001 | .098 | .082 | .201 | .057 | .408 | .071 | .024 |

b) Incident Mode 9 (main Lg arrival)

| Relative Mode Number: | | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|
| Model: | C | .006 | .014 | .006 | .336 | .360 | .240 | .010 | .000 | .000 |
| | D | .006 | .017 | .006 | .336 | .348 | .250 | .012 | .000 | .000 |
| | E | .008 | .023 | .144 | .090 | .423 | .160 | .078 | .004 | .000 |
| | F | .078 | .102 | .003 | .303 | .260 | .053 | .068 | .048 | .026 |

c) Incident Mode 14 (coda of Lg)

| Relative Mode Number: | | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|
| Model: | C | .000 | .000 | .001 | .032 | .846 | .090 | .001 | .000 | .000 |
| | D | .000 | .000 | .001 | .029 | .828 | .109 | .001 | .000 | .000 |
| | E | .000 | .000 | .020 | .014 | .397 | .476 | .073 | .001 | .005 |
| | F | .001 | .008 | .006 | .084 | .672 | .109 | .006 | .020 | .014 |

For incident modes 4 and 9, the nature of the energy spread induced across the modes of the reference structure has been to principally couple Lg type modes together and so the pattern of energy spread shows a skew towards higher mode numbers for mode 4 and lower numbers for mode 9. Whereas, for incident mode 14, the coupling is largely into a the immediate neighbouring modes with similar character and also into the Sn modes.

We should note that even for the moderately heterogeneous model D (fig 3) there has been a significant modification of the distribution of energy between modes after passage through 1000 km of structure. Such modifications over a band of frequencies will lead to theoretical seismograms which will differ significantly from the predictions for the horizontally stratified reference model.

*Studies of Three-dimensional Heterogeneity*

In addition to the work on two-dimensional structures, efforts have been made over the past year to extend the mode coupling techniques to three-dimensional models. For relatively weak heterogeneity it has proved possible to construct a approximate development which should be able to give a good represenation of the Lg wave field when most significant energy travels close to a direct path between source and receiver, and so multipathing is not too important. This approach will require the solution of partial differential equations for the coupled modes in a spatial swath about the direct path and both Love and Rayleigh wave modes will need to be considered at the same time (cf Snieder 1986). Thus even at 1 Hz at least 30 coupled partial differential equations will be required. The number of modes needed at each frequency will be at least double that for the two-dimensional case and also now calculations will need to be carried for a number of offset points in the transverse direction to the path. This requirements will place heavy demands on computational resources in future work and as a result considerable effort is being expended on the design of suitable algorithms for the calculations.

## DISCUSSION

The current research effort has been directed towards determining the numerical characteristics of the Kennett (1984) mode-coupling scheme for guided wave propagation over extended distances at frequencies of 1 Hz and above. The existing models are two-dimensional and calculations are carried out for one frequency at a time.

The results of the studies of heterogeneous crustal models show that reflection from distributed heterogeneity is not very important. However, both reflection and transmission processes should be included, if at all possible, in the numerical calculations in order to ensure the accuracy of estimates of modal transmission. The coupled mode technique works well with heterogeneous models which differ from a stratified reference model by up to 2 per cent with a limited bandwidth of interaction between modes. The approach can be used for higher levels of heterogeneity but a large number of modes needs to be considered with consequent high computation costs.

One of the major effects of crustal heterogeneity is to introduce the possibility of smearing out the main amplitude peak in the Lg wave train over a band of group velocities. As a result, an effective measure of the energy content of the Lg waves will be to consider the integrated amplitude along the traces beween group velocities of 3.6 and 3.3 km/s. The influence of mantle heterogeneity on Lg wave propagation was found to be quite small. However, any conversion into Sn type modes will give arrivals which will have apparent group velocities of between 4.2 and 3.7 km/s, depending on the position where conversion occurs. Such arrivals will appear in a portion of the seismic record which would be predicted to be very quiet in stratified medium calculations, and as a result may appear to be more prominent than expected.

The costs of performing the coupled mode calculations, at fixed step length in the horizontal direction, would rise as the square of the number of modes employed (and therefore approximately as the frequency) if full modal coupling is allowed. The introduction of a limited bandwidth of interaction reduces the cost but still it will be difficult to push the calculations to very high frequency. At 2 Hz, about 35 modes are needed for a typical continental model to give an adequate representation of the effects of heterogeneity on either Love or Rayleigh waves, and this would increase to about 70 modes for 4 Hz propagation. As the wavelength reduces the horizontal step length in the solution of the differential equations needs to be reduced to maintain accuracy. This gives a further increase in computational cost approximately linear in frequency. The numerical algorithms used for the solution of the matrix Ricatti equations for reflection and transmission work well with large numbers of modes, but the differential equations are non-linear and there is the possibility of loss of precision if very large numbers of coupled equations are considered.

Nevertheless the coupled mode approach to the calculation of guided wave propagation has the potential to enable the character of Lg wave propagation to be determined for classes of model which cannot easily be treated by any other means. Both deterministic and statistical models of heterogeneity can readily be incorporated into the computation scheme. The coupled mode scheme does therefore allow an assessment of structural effects on regional phase propagation which is complementary to existing techniques using horizontally stratified models.

The next phase of work will be directed towards using the experience from the two-dimensional models to design a computational scheme for mode-coupling in three-dimensions for both Love and Rayleigh waves. This scheme will take advantage of approximations such as limited bandwidth of interaction to reduce total computation time. We will also pursue the problem of generating effective and consistent theoretical seismograms by combining mode-coupling results from a number of frequencies.

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## APPENDIX: Modal evolution equations

### *1. Two-dimensional problems*

We present here a brief derivation of the coupled mode technique introduced by Kennett (1984).

We will firstly restrict attention to a two-dimensionally heterogeneous structure and work with a fully anisotropic medium for which a very compact notation is available. The heterogeneity is assumed to be invariant in the transverse direction to the wave vector $\underset{\sim}{k}$, and so the direction of this vector will remain constant even in the anisotropic case where the ray direction associated with a particular mode does not lie along $\underset{\sim}{k}$.

We consider the heterogeneity as a deviation from the properties of a reference model. This perturbation is not necessarily small but must be bounded in space. We seek a representation of the displacement field in terms of a superposition of the modal eigenfunctions of the reference model with modal coefficients which vary with position.

We work in a cartesian coordinate system, with propagation in the $x-z$ plane and introduce the displacement vector $\underset{\sim}{w} = (w_x, w_y, w_z)$. We also need to specify the traction field. In the case of a modal wavetrain propagating in the $x$ -direction we are concerned with continuity of traction on vertical planes and so we take $\underset{\sim}{t}_1 = (\tau_{11}, \tau_{12}, \tau_{13})$. In addition the material continuity requirements mean that we also need to be able to refer to the traction on a horizontal plane $\underset{\sim}{t}_3 = (\tau_{31}, \tau_{32}, \tau_{33})$. For the 2-D situation we need a number of combinations of elastic moduli which may be represented compactly in terms of matrices $\mathbf{C}_{ij}$ such that

$$(\mathbf{C}_{ij})_{kl} = c_{kilj}$$

where the $c_{kilj}$ are the anisotropic moduli.

The equations of motion and the stress-strain conditions can be cast into a form where the derivatives with respect to the propagation direction $x$ appear only on the left hand side of the equations

$$\frac{\partial}{\partial x}\begin{pmatrix} \underset{\sim}{w} \\ \underset{\sim}{t}_1 \end{pmatrix} = \begin{pmatrix} A_{ww} & A_{wt} \\ A_{tw} & A_{tt} \end{pmatrix}\begin{pmatrix} \underset{\sim}{w} \\ \underset{\sim}{t}_1 \end{pmatrix} - \begin{pmatrix} 0 \\ \underset{\sim}{f} \end{pmatrix}, \qquad (1)$$

in the presence of a volume force contribution $\underset{\sim}{f}$.
The differential operators $A_{..}$ do not depend on the horizontal derivatives of the material properties and for the 2-D case have the explicit form

$$\begin{pmatrix} A_{ww} & A_{wt} \\ A_{tw} & A_{tt} \end{pmatrix} = \begin{pmatrix} -C_{11}^{-1} C_{13}\partial_z & C_{11}^{-1} \\ -\rho\omega^2 I - \partial_z [Q_{33}\partial_z & -\partial_z [C_{31} C_{11}^{-1} \end{pmatrix}, \qquad (2)$$

where we have written $\partial_z$ for $\partial/\partial z$ and set

$$Q_{33} = C_{33} - C_{31} C_{11}^{-1} C_{13} \qquad (3)$$

The unclosed brackets in (2) indicate that the operator acts to its right. Across an interface $z = const$, the $\tau_{13}$ component of

will be continuous, but $\tau_{11}$, $\tau_{12}$ are not required to be continuous. On the other hand $\underset{\sim}{t}_3$ will be continuous and can be represented in terms of $\underset{\sim}{w}$, and $\underset{\sim}{t}_1$ as

$$\underset{\sim}{t}_3 = Q_{33}\,\partial_z \underset{\sim}{w} + C_{31} C_{11}^{-1} \underset{\sim}{t}_1 . \tag{4}$$

The equations (1)-(3) apply in an arbitrarily heterogeneous medium but by themselves provide no direct link to the surface wave case we wish to consider.

When the heterogeneous medium does not deviate too strongly from the reference medium, we can envisage that it may be possible to find a representation in terms of the surface wave modes of the reference structure. We here consider the case of a fixed, stratified, reference medium as used in the examples of Kennett (1984) and Kennett & Mykkeltveit (1984).

At each position $x$ we consider cutting the actual structure along a vertical plane and then weld on the reference structure to ensure continuity of displacement $\underset{\sim}{w}$ and traction $\underset{\sim}{t}_1$. In this reference structure we can now represent the displacement and traction at $x$ as a superposition of modal contributions and write

$$\begin{pmatrix} \underset{\sim}{w}(x) \\ \underset{\sim}{t}_1(x) \end{pmatrix} = \sum_r c_r(x) \exp(i k_r x) \begin{pmatrix} \underset{\sim}{w}_r^{\circ}(k_r, z) \\ \underset{\sim}{t}_{1r}^{\circ}(k_r, z) \end{pmatrix}, \tag{5}$$

where the $\underset{\sim}{w}_r^{\circ}$ are the displacement eigenfunctions for the reference medium and the horizontal tractions $\underset{\sim}{t}_{1r}^{\circ}$ are derived from $\underset{\sim}{w}_r^{\circ}$. The sum is to be taken over all relevant modes at frequency $\omega$. In order to give a full representation of the displacement this sum will normally involve both forward and backward travelling waves (i.e. both positive and negative $k_r$). As discussed by Kennett (1984) we can arrange the deepest part of the model of the reference medium so that (5) is a sum over an infinite set of orthogonal modes, and achieve a complete representation of the seismic wavefield. For example, the constructive interference of sufficient numbers of surface wave modes will synthesise body wave phases (see e.g. the examples in chapter 11 of Kennett (1983)). However, especially at higher frequencies, this requires a very large number of modes. In the varying medium we wish to concentrate on the surface wave field and so would like to be able to work in terms of a restricted (and not too large) set of modes. Such a truncation of the expansion (5) imposes limitations on the size and character of the heterogeneity which can be tackled.

We recall that $\tau_{11}$, $\tau_{12}$ are, in general, not continuous across horizontal interfaces and so if the material discontinuities in the heterogeneous model do not coincide with those in the reference model, we have the problem of representing a jump in $\underset{\sim}{t}_1$ across an interface by a superposition of continuous traction vectors $\underset{\sim}{t}_{1r}^{\circ}$. A satisfactory fit can be achieved with many modes, but the accuracy of the representation may be limited with a restricted mode set (cf. Fourier series). As a result it is desirable that the major discontinuities in both the heterogeneous and reference models should be coincident.

In the reference structure the modal contributions (5) satisfy the coupled equations

$$\sum_r ik_r \hat{c}_r \begin{pmatrix} \underset{\sim}{W}_r^\circ \\ \underset{\sim}{t}_{1r}^\circ \end{pmatrix} - A^\circ \sum_r \hat{c}_r \begin{pmatrix} \underset{\sim}{W}_r^\circ \\ \underset{\sim}{t}_{1r}^\circ \end{pmatrix} = 0, \tag{6}$$

in the absence of any applied force, with coefficients $\hat{c}_r = c_r \exp(ik_r x)$. We have written $A^\circ$ to indicate the form of the differential operators for the reference medium.

In a laterally varying medium we have to require the modal coefficients to vary with position. On expanding the differential operators $A$ as $A^\circ + \Delta A$, to separate off the reference medium contribution, we find

$$\sum_r \left\{ ik_r \hat{c}_r \begin{pmatrix} \underset{\sim}{W}_r^\circ \\ \underset{\sim}{t}_{1r}^\circ \end{pmatrix} + \frac{\partial \hat{c}_r}{\partial x} \begin{pmatrix} \underset{\sim}{W}_r^\circ \\ \underset{\sim}{t}_{1r}^\circ \end{pmatrix} \right\} = (A^\circ + \Delta A) \sum_r \hat{c}_r \begin{pmatrix} \underset{\sim}{W}_r^\circ \\ \underset{\sim}{t}_{1r}^\circ \end{pmatrix}. \tag{7}$$

These equations may be rewritten as

$$\sum_r (ik_r - A^\circ) \hat{c}_r \begin{pmatrix} \underset{\sim}{W}_r^\circ \\ \underset{\sim}{t}_{1r}^\circ \end{pmatrix} = - \sum_r \frac{\partial \hat{c}_r}{\partial x} \begin{pmatrix} \underset{\sim}{W}_r^\circ \\ \underset{\sim}{t}_{1r}^\circ \end{pmatrix} + \Delta A \sum_r \hat{c}_r \begin{pmatrix} \underset{\sim}{W}_r^\circ \\ \underset{\sim}{t}_{1r}^\circ \end{pmatrix}, \tag{8}$$

in which we can recognise the left hand side as having the same form as in (6), but we have in addition, on the right side, a term which is equivalent to a generalised volume force applied to the reference structure.

As well as the equation of motion we require the wavefield to satisfy certain continuity conditions at internal amd external boundaries where the elastic moduli are discontinuous. The most general case of an interface in the heterogeneous medium will be titled and displaced from those of the reference medium. Continuity of displacement is assured at any interface from the continuity properties of the displacement eigenfunctions. We also require that the tractions normal to any interface should be continuous. We will assume that all such surfaces do not vary rapidly in the horizontal direction. Then for an interface in the laterally varying medium described by the function $h(x)$, we may work to first order in the slope ( $\dot{h}$ ), and require

$$\left[ (\dot{h} I + C_{31} C_{11}^{-1}) \sum_r \hat{c}_r(x) \underset{\sim}{t}_{1r}^\circ + Q_{33} \sum_r \hat{c}_r(x) \partial_z \underset{\sim}{W}_r^\circ \right] = 0 \tag{9}$$

where $I$ denotes the identity matrix and the square brackets indicate the jump in the enclosed quantities across the interface, evaluated from bottom to top.

If we now separate out the contribution from the properties of the reference medium from the local variations, we may write

$$\left[ (C_{31} C_{11}^{-1})^\circ \sum_r \hat{c}_r(x) \underset{\sim}{t}_{1r}^\circ + Q_{33}^\circ \sum_r \hat{c}_r(x) \partial_z \underset{\sim}{W}_r^\circ \right]$$

$$= - \left[ \{ \Delta (C_{31} C_{11}^{-1}) + \dot{h} I \} \sum_r \hat{c}_r(x) \underset{\sim}{t}_{1r}^\circ + \Delta Q_{33} \sum_r \hat{c}_r(x) \partial_z \underset{\sim}{W}_r^\circ \right. \tag{10}$$

The difference terms such as $\Delta (C_{31} C_{11}^{-1})$ in (10) indicate the discrepancy between the combinations of elastic moduli for the

heterogeneous and reference models. In the absence of any lateral heterogeneity the expression on the left hand side of (10) would vanish. The presence of the heterogeneity is thus equivalent to inducing a traction discontinuity into the reference medium, along the line of the interface. Such a traction discontinuity is equivalent to a localised volume force along the interface

$$\underset{\sim}{f}^{T}(x) = -[\underset{\sim}{T}]\,\delta(z-h(x))$$

(see e.g. section 3.1 of Aki & Richards (1980)).

We can therefore represent the combination of the equations of motion and the boundary conditions in the laterally varying medium by means of the following system of equations

$$\sum_r \hat{c}_r\,(ik_r - A^0)\begin{pmatrix}\underset{\sim}{W}_r^0\\ \underset{\sim}{t}_{1r}^0\end{pmatrix} = \sum_r\left\{-\frac{\partial\hat{c}_r}{\partial x}\begin{pmatrix}\underset{\sim}{W}_r^0\\ \underset{\sim}{t}_{1r}^0\end{pmatrix} + \hat{c}_r\Delta A\begin{pmatrix}\underset{\sim}{W}_r^0\\ \underset{\sim}{t}_{1r}^0\end{pmatrix} - \begin{pmatrix}0\\ \underset{\sim}{f}_r\end{pmatrix}\right\}, \tag{11}$$

where the force term is a summation of interface contributions

$$\underset{\sim}{f}_r = \sum_n \left[\{\Delta(C_{31}C_{11}^{-1}) + \dot{h}_n I\}\underset{\sim}{t}_{1r}^0 + \Delta Q_{33}\partial_z\underset{\sim}{W}_r^0\right]_n \delta(z-h_n(x))$$

With the abstraction of this force contribution the interface conditions on the wavefield are reduced to

$$\left[\sum_r \hat{c}_r(x)\,\underset{\sim}{W}_r^0\right]_n = 0, \quad \left[\sum_r \hat{c}_r(x)\{(C_{31}C_{11}^{-1})^0\underset{\sim}{t}_{1r}^0 + Q_{33}^0\partial_z\underset{\sim}{W}_r^0\}\right]_n = 0.$$

where the subscript $n$ indicates the jump at any interface. These constraints will be automatically satisfied by the representation (5). Equation (11) can be further simplified because the modal eigenfunctions are just the free solutions for the reference structure and so the left hand side of the equation vanishes. This leaves us with

$$\sum_r \frac{\partial\hat{c}_r}{\partial x}\begin{pmatrix}\underset{\sim}{W}_r^0\\ \underset{\sim}{t}_{1r}^0\end{pmatrix} = \sum_r \hat{c}_r\left\{\Delta A\begin{pmatrix}\underset{\sim}{W}_r^0\\ \underset{\sim}{t}_{1r}^0\end{pmatrix} - \begin{pmatrix}0\\ \underset{\sim}{f}_r\end{pmatrix}\right\}. \tag{12}$$

We now exploit the orthogonality relation between different modal eigenfunctions for the reference medium

$$i\int_0^\infty dz\,\{\underset{\sim}{W}_i^0(-k_i,z)\cdot\underset{\sim}{t}_{1j}^0(k_j,z) - \underset{\sim}{t}_{1i}^0(-k_i,z)\,\underset{\sim}{W}_j^0(k_j,z)\} = \delta_{ij} \tag{13}$$

to get a set of coupled first order differential equations for the modal coefficients $\{c_r\}$, appropriate for both elastic and anelastic heterogeneity. These equations can be written in the form

$$\begin{aligned}\frac{\partial\hat{c}_q}{\partial x} = i\sum_r \hat{c}_r\int_0^\infty dz\,\{\,&\bar{\underset{\sim}{t}}_{1q}^0\cdot[\Delta C_{11}^{-1}\underset{\sim}{t}_{1r}^0 - \Delta(C_{11}^{-1}C_{13})\partial_z\underset{\sim}{W}_r^0]\\ &-\bar{\underset{\sim}{W}}_q^0\cdot[-\Delta\rho\omega^2\underset{\sim}{W}_r^0 - \partial_z\{\Delta Q_{33}\partial_z\underset{\sim}{W}_r^0 + \Delta(C_{31}C_{11}^{-1})\underset{\sim}{t}_{1r}^0\}]\}\\ &+ i\sum_n\left[\bar{\underset{\sim}{W}}_q^0\cdot\sum_r \hat{c}_r(\dot{h}_n\underset{\sim}{t}_{1r}^0 + \Delta Q_{33}\partial_z\underset{\sim}{W}_r^0 + \Delta(C_{31}C_{11}^{-1})\underset{\sim}{t}_{1r}^0)\right]_n,\end{aligned} \tag{14}$$

where we have written

$$\bar{\underset{\sim}{W}}_q^0 = \underset{\sim}{W}_q^0(-k_q,z).$$

Equation (14) can be simplified by integration by parts, to yield

$$\frac{\partial \hat{c}_q}{\partial x} = i \sum_r \hat{c}_r \int_0^\infty dz \, \{ \bar{\mathbf{t}}_{1q}^{\circ} \cdot \Delta C_{11}^{-1} \mathbf{t}_{1r}^{\circ} + \Delta\rho\omega^2 \bar{\mathbf{W}}_q^{\circ} \cdot \mathbf{W}_r^{\circ} - \bar{\mathbf{t}}_{1q}^{\circ} \cdot \Delta(C_{11}^{-1} C_{13}) \partial_z \mathbf{W}_r^{\circ} - \partial_z \bar{\mathbf{W}}_q^{\circ} \{ [\Delta Q_{33} \partial_z \mathbf{W}_r^{\circ} + \Delta(C_{31} C_{11}^{-1}) \mathbf{t}_{1r}^{\circ} ] \} + i \sum_r \hat{c}_r \sum_n \dot{h}_n [ \bar{\mathbf{W}}_q^{\circ} \cdot \mathbf{t}_{1r}^{\circ} ]_n \qquad (15)$$

In which we see that there is no explicit interface term in the case of a flat boundary ( $\dot{h}_n = 0$ ). However, the modification of an interface will appear through the integral term in (15), where the properties of the laterally varying medium differ from the reference and there will be a significant contribution to the integral which will vary with horizontal position. In a first-order perturbation theory the whole of this effect would be projected onto a specific interface term, but here we are able to make a more detailed allowance for the behaviour.

The combination of the modal representation (5) with the imposition of continuity of traction at each interface thus leads to the set of coupled equations

$$\frac{\partial}{\partial x} c_q = i \sum_r K_{qr} \exp(-ik_q x) \exp(ik_r x) c_r \qquad (16)$$

for the modal coefficients, where the coupling coefficients between the modes can be written as

$$K_{qr} = \int_0^\infty dz \, \{ \bar{\mathbf{t}}_{1q}^{\circ} \cdot \Delta C_{11}^{-1} \mathbf{t}_{1r}^{\circ} + \Delta\rho\omega^2 \bar{\mathbf{W}}_q^{\circ} \cdot \bar{\mathbf{W}}_r^{\circ} - \partial_z \bar{\mathbf{W}}_q^{\circ} \Delta Q_{33} \partial_z \mathbf{W}_r^{\circ} - \partial_z \bar{\mathbf{W}}_q^{\circ} \Delta(C_{31} C_{11}^{-1}) \mathbf{t}_{1r}^{\circ} - \bar{\mathbf{t}}_{1q}^{\circ} \cdot \Delta(C_{11}^{-1} C_{13}) \partial_z \mathbf{W}_r^{\circ} \} + \sum_n \dot{h}_n [ \bar{\mathbf{W}}_q^{\circ} \cdot \mathbf{t}_{1r}^{\circ} ]_n \qquad (17)$$

The coupled first-order equations (16) are not very easy to solve because we have a two-point boundary value problem with both reflected and transmitted modes to be determined. Kennett (1984, section 3) has shown that an effective procedure is to work with the reflection and transmission matrices for a sequence of models encompassing increasing portions of the heterogeneous medium. This leads to an initial value problem for two coupled matrix Ricatti equations. These nonlinear differential equations for the reflection and transmission matrices can be readily solved numerically.

## 2 *Three-dimensional Problems*

In a medium whose properties vary in three dimensions, it is possible to generate evolution equations for the displacement and horizontal tractions with respect to the horizontal coordinates. The analysis parallels the two-dimensional case, but now one has to take account of variation, in both the material properties and the wavefield, transverse to the sirection of current interest.

If we concentrate on the evolution in the $x$ -direction we find

$$\frac{\partial}{\partial x}\begin{pmatrix} \underset{\sim}{W} \\ \underset{\sim}{t}_1 \end{pmatrix} = \begin{pmatrix} -C_{11}^{-1}C_{13}\partial_z - C_{11}^{-1}C_{12}\partial_y & C_{11}^{-1} \\ \begin{array}{l} -\rho\omega^2 I - \partial_z [Q_{33}\partial_z \\ -\partial_z [Q_{32}\partial_y - \partial_y [Q_{23}\partial_z - \partial_y [Q_{22}\partial_y \end{array} & \begin{array}{l} \partial_z [C_{31}C_{11}^{-1} \\ +\partial_y [C_{21}C_{11}^{-1} \end{array} \end{pmatrix} \begin{pmatrix} \underset{\sim}{W} \\ \underset{\sim}{t}_1 \end{pmatrix}, \tag{18}$$

where
$$Q_{22} = C_{22} - C_{21}C_{11}^{-1}C_{12}.$$
$$Q_{23} = C_{23} - C_{21}C_{11}^{-1}C_{13}, \quad Q_{32} = C_{32} - C_{31}C_{11}^{-1}C_{12}.$$

In equation (18), the terms have been written in an order which emphasises those contributions which appear in the two-dimensional formulation. Previously we were able to eliminate the dependence on depth $z$ and convert from evolution equations in displacement and traction to equations in the modal expansion coefficients by making use of the modal orthogonality properties for modes travelling in the same (or opposite) directions. With 3-D heterogeneity we now have the possibility of modes travelling in different directions and also coupling between Love and Rayleigh modes. Unfortunately, we cannot write down any simple form for the representation of the wavefield as a sum of modal contributions.

If we consider a situation where the heterogeneity is not too strong and concentrate attention on propagation dominantly along the $x$ direction, we can generate a useful approximation by neglecting the angular dependence of modal interaction. By this means, we obtain a generalisation of the 2-D equations with a weak dependence on the transverse ( $y$ ) direction. The partial differential equations in $x$ for the modal coefficients are then to be integrated over a swath surrounding the direct propagation path from source to receiver.

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